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UCRL-JRNL-201347

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December 8, 2003

Physics of Plasmas

Disclaimer

UCRL-JRNL-201347

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Abstract

In some plasma confinement systems (e.g., field-reversed configurations and levitated dipoles) the confinement is provided by a closed-field-line poloidal magnetic field. We consider the influence of the magnetic field perturbations on the structure of the magnetic field in such systems and find that the effect of perturbations is quite different from that in the systems with a substantial toroidal field. In particular, even infinitesimal perturbations can, in principle, lead to large radial excursions of the field lines in FRCs and levitated dipoles. Under such circumstances, particle drifts and particle collisions may give rise to significant neoclassical transport. Introduction of a weak regular toroidal magnetic field reduces radial excursions of the field lines and neoclassical transport.

PACS Numbers: 52.25.Fi, 52.55.Hc, 52.55.Lf

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There exists a class of plasma confinement systems where the field lines of the confining magnetic field are closed, but there is no toroidal magnetic field. The examples include levitated dipoles (e.g., [1] and references therein) and field-reversed configurations (FRC) (e.g., [2] and references therein). Diffuse pinches and inverse pinches without axial magnetic field (e.g., [3, 4]) also belong to this class, provided they are long enough not to be governed by end effects.

It turns out that the effect of magnetic field perturbations on the geometry of the magnetic field lines in such systems has interesting features directly related to the absence of the toroidal magnetic field. In particular, radial excursions of the perturbed magnetic field lines can be large even for very small perturbations. In this brief communication, we provide a set of equations describing this effect, discuss particle trajectories and collision-induced neoclassical transport in the perturbed field, and evaluate the magnitude of the toroidal magnetic field needed to substantially reduce the transport.

We use coordinate frame defined by the unperturbed magnetic field: the poloidal flux Φ, the distance l measured along the unperturbed field line from the equator (Fig. 1a), and the toroidal angle φ. The unperturbed field lines form closed curves in the poloidal plane. We do not make any assumptions about the plasma beta, so that the unperturbed field lines are determined by both the external currents and equilibrium (i.e., purely toroidal and axisymmetric) plasma currents.

We decompose magnetic field perurbations over three mutually perpendicular directions: the direction normal to the flux surface ("radial"), the direction tangential to the flux surface and lying in the poloidal plane, and the toroidal direction. We denote these components by δB_n, δB_p, δB_t, respectively. In the linear (over δB) approximation, the deviation of the field line from the initial one in the normal and toroidal directions can be represented as:

$$\frac{d\Phi}{dl} = 2\pi R\delta B_n; \quad \frac{d\varphi}{dl} = \frac{\delta B_t}{B_0 R} \tag{1}$$

The (small) variation of Φ and φ during one full turn in the poloidal direction is:

$$\Delta\Phi = 2\pi\oint R\delta B_n dl \tag{2}$$

$$\Delta\varphi = \oint \frac{\delta B_t dl}{RB_0} \tag{3}$$

By marking successive intersections of the field line with the equatorial plane, after every full turn in the poloidal direction, one obtains a puncture plot, which is quasi-continuous because of the assumed smallness of perturbations. Dividing $\Delta\Phi$ by $\Delta\varphi$ and replacing finite differences by differentials, one gets an equation for this (quasicontinuous) curve:

$$\frac{d\Phi}{d\varphi} = \frac{2\pi\oint R\delta B_n dl}{\oint \frac{\delta B_t dl}{RB_0}} \equiv F(\Phi,\varphi) \tag{4}$$

The numerator and denominator of this equation are functions of Φ and φ.Their dependence on φ is periodic, with the period 2π.

The integration over l in (2) can be considered as a kind of a “projection” operation. In other words, not all types of perturbations contribute to the wandering of the field lines. In particular, one can show that $\Delta\Phi$ (as well as $\Delta\varphi$) is identically zero if perturbations are of an ideal MHD type, where the line-tying constraint is satisfied exactly (i.e., $\delta\mathbf{B} = \nabla\times\xi\times\mathbf{B}$ where $\boldsymbol{\xi}$ is the displacement of a fluid element. This notion is in agreement with the general observation that, in the ideal MHD, perturbations cannot change the topology of the field lines.

Two more examples: If a uniform axial magnetic field is imposed on the system shown on Fig. 1a, $\Delta\Phi$ is again zero, because the r.h.s. in Eq. (2) corresponds to the magnetic flux of a uniform field through a closed toroidal surface (which flux is, of course, zero). For the systems with the up-down symmetry (in the unperturbed state), the condition $\Delta\Phi$=0 is satisfied if the perturbation is antisymmetric with respect to the equatorial plane. This circumstance was noticed and used in Ref. [5] to minimize a negative effect of RF current drive on the plasma confinement in the RF-driven FRC. So, not all types of magnetic perturbations cause radial wandering of the field lines.

Equation (4) is quite general in that it is not based on any assumptions about the plasma beta. It is equally applicable to the levitated dipoles and FRCs. In the case of a diffuse pinch, one has to introduce an axial coordinate ζ instead of φ, and understand by Φ the flux per unit length of the pinch. The resulting equation equivalent to (4) then becomes:

$$\frac{d\Phi}{d\varsigma} = B_0(\Phi)\frac{\oint \delta B_n dl}{\oint \delta B_t dl} \tag{5}$$

In this case, as the unperturbed field lines are just circles, only poloidally-symmetric part of the perturbation contributes to displacements.

For a given unperturbed magnetic field, and for the known perturbations, the right hand side of Eq. (4) is a known function. Integrating Eq. (4), one finds the puncture plot and excursions of the field lines over the flux coordinate. Remarkably, these excursions do not depend on the amplitude of perturbations, only on their spatial structure. In other words, if one multiplies both δB_n and δB_t by the same constant factor, equations (4) and (5) do not change.

In the most general case, by using the Fourier decomposition of the perturbations over the toroidal angle, one can represent the numerator (denominator) of the r.h.s. of Eq. (4) as $\sum_{m=1}^{\infty} C_m^{(1,2)}(\Phi)\cos m\varphi + S_m^{(1,2)}(\Phi)\sin m\varphi$, where subscript "1" ("2") refers to the numerator (denominator). The coefficients *C* and *S* are determined by the radial structure of the perturbation. If one toroidal mode (say, the *m*th mode) of the perturbation is strongly dominant, one gets

$$F(\Phi,\varphi) = \frac{C_m^{(1)}(\Phi)\cos m\varphi + S_m^{(1)}(\Phi)\sin m\varphi}{C_m^{(2)}(\Phi)\cos m\varphi + S_m^{(2)}(\Phi)\sin m\varphi} \qquad (6)$$

Eq. (4) with *F* as in Eq. (6) may have singular points, where both numerator and denominator turn zero. Assuming that those are simple zeros, one finds a chain of elliptic and hyperbolic singular points, determining a string of islands (Fig. 1b). This may happen when instabilities create non-axisymmetric perturbed currents within the plasma. What is different from "traditional" islands found in, e.g., tokamaks, is that the "thickness" of the islands is determined now solely by the spatial structure of perturbation, not by its amplitude. Note also that Fig. 1b depicts equatorial cross-section z=0 (not the poloidal cross-section φ=const normally used in tokamak surface-of-section plots). If the perturbation encompasses the whole plasma radius, the island thickness is of order of plasma radius, even for very small perturbations; however, the number of transits (or, equivalently, the field line length) required for a field-line to circulate around the island is inversely proportional to the perturbation amplitude. Roughly speaking, for the field line to be displaced radially by the characteristic size h of order of a radial length-scale of perturbations, the field line has to make a path

$$s \sim h \frac{B_0}{\delta B} \tag{7}$$

A special class of perturbations is a perturbation of the form of a uniform magnetic field perpendicular to the axis of the system. [As we have already mentioned, a uniform *axial* magnetic field does not contribute to the field line wandering.] We consider its effect on the example of a long ($L>>a$) racetrack-shaped FRC (Fig. 2). Assume that perturbing field is directed along the axis x, i.e.,

$$\delta B_n = b\cos\varphi; \ \ \delta B_t = -b\sin\varphi \tag{8}$$

where b is the magnetic field strength of the perturbation.

In the $L>>a$ case, the constancy of the plasma pressure along the field line, combined with the condition of the radial equilibrium means that the magnetic field strength on the outer and inner sections of some field line is the same. We shall mark the unperturbed magnetic field strength by its magnitude at the outer part of FRC, i.e., we specify the function $B(r_e)$. The condition that the magnetic flux comprised between two neighboring flux surfaces is constant, yields an equation (e.g., [2]):

$$r_e^2 + r_i^2 = a^2 \tag{9}$$

In the case of a long racetrack, one can neglect the contribution of the end regions to the integrals in Eq. (3). Then, the integral over the field line is approximately equal to contributions of the straight segments, whose length is approximately equal to the FRC length L. Specifically,

$$\oint R\delta B_n dl = Lb(r_e - r_i)\cos\varphi, \tag{10}$$

and

$$\oint \frac{\delta B_t dl}{RB_0} = -\frac{Lb}{B_0(r_e)}\left(\frac{1}{r_i}+\frac{1}{r_e}\right)\sin\varphi \tag{11}$$

One obviously has

$$\frac{d\Phi}{dr_e} = 2\pi B_0(r_e)r_e \tag{12}$$

Eq. (4) then yields:

$$\frac{dr_e}{d\varphi} = -\frac{r_e - r_i}{r_e + r_i} r_i \cot\varphi \tag{13}$$

where r_i is related to r_e by Eq. (9). An elementary integration yields a family of puncture plots:

$$r_e - \sqrt{a^2 - r_e^2} = \frac{C}{|\sin\varphi|} \tag{14}$$

The constant C satisfies inequalities: $0<C<a$. A set of plots is shown on Fig. 2b. Note a great degree of universality of this result: it does not depend on the details of the pressure distribution over the flux surfaces.

Now we discuss possible effect of magnetic field perturbations on particle confinement, which is determined by particle trajectories, not just by the magnetic field line path. Two effects are of importance here: mirror reflection of the particles from the zones of a strong field, and particle drifts. We make the further estimates for the levitated dipole. Particles with a sufficiently large pitch angle experience mirror reflection from the zone of a strong magnetic field inside the ring. These particles are bouncing along a segment of a field line between the turning points. They do not cover the whole field line and do not execute large radial excursions shown on Fig. 1b. Conversely, transit particles would cover the whole field line and would suffer from large radial excursions.

Consider now effects of particle drift. If the toroidal drift velocity v_d is large, so that toroidal displacement caused by this drift within one poloidal period of the particle motion exceeds toroidal displacement $\Delta\varphi$ [Eq. (3)] caused by the presence of a perturbing magnetic field, radial excursions of particles decrease compared to the global scale of the system. The criterion for this to happen reads as

$$\frac{r_C}{a} > \frac{\delta B}{B_0} \tag{15}$$

where r_C is the cyclotron radius of a particle. Clearly, this condition is most restrictive for the electrons. Taking a=500 cm, T_e~30 keV, and the magnetic field B_0~1 T, on finds that the relative value of perturbations must be small, $\delta B/B_0 < 10^{-4}$.

The transiting particle during one full circle of the poloidal motion gets displaced in the flux coordinate by the amount $\Delta\Phi$ determined by Eq. (2). At the same time, under condition (15), it drifts in the toroidal direction by an angle

$$\Delta\varphi = \oint \frac{v_d\, dl}{v_{\parallel} R} \tag{16}$$

Accordingly, an equation for the projection of the particle trajectory on the equatorial plane will be (Cf. Eq. (4)):

$$\frac{d\Phi}{d\varphi} = \frac{2\pi \oint R\delta B_n dl}{\oint \frac{v_d\, dl}{v_{\parallel} R}} \tag{17}$$

The denominator here does not depend on the toroidal angle; accordingly Eq. 17 implies, for example, that field-errors with m = 1 produce a horizontal shift, δr, of the normally circular drift orbits and those with m = 2 become elliptical , with the parameters of orbits in both cases depending on the particle integrals of motion.

Let's evaluate radial excursions δr of transiting particles. First, we note that, in the case of a levitated dipole, $R \sim a$, so that $\delta\Phi \sim 2\pi a B_0 \delta r$. Then, we estimate v_d as $v_T(r_C/a)$, and $v_{\|}$ as v_T. With that, Eq. (17) yields for the global-scale perturbations (i.e., for perturbations with a spatial scale $\sim a$)

$$\delta r \sim \frac{a}{m}\frac{a\delta B}{r_C B_0} \tag{18}$$

Assuming that the magnetic field in the center of the ring is 10 T, one can evaluate the number of transiting particles as $\varepsilon \sim 1/10$. The time for one radial excursion is (under condition (15)) $\sim a^2/mr_C v_{Te}$ and, for plasma densities $\sim 10^{14}$ cm^{-3}, is shorter than the electron scattering time over the loss-cone angle, $\tau \sim \varepsilon/\nu_e$. Therefore, the corresponding neoclassical electron thermal diffusivity is:

$$\chi \sim \varepsilon\frac{\delta r^2}{\tau} \sim \nu_e \frac{a^2}{m^2}\left(\frac{a\delta B}{r_C B_0}\right)^2 \tag{19}$$

The corresponding electron heat loss time would be $\sim (m^2/\nu_e)\left(r_C B_0/a\delta B\right)^2$. If the inequality (15) holds by a 10-fold margin, *i.e.*, the relative perturbation level is $\sim 10^{-5}$, the confinement time for large-scale error modes, $m = 1$, turns out to be ~100 electron scattering times. In order for the electron neoclassical thermal diffusivity to be the same order as classical ion diffusivity, the field errors must be small, $\delta B/B_0 < 0.7m \times 10^{-7}$. [Estimate (19) corresponds to a so-called "banana" regime; for lower plasma temperatures, when the scattering time becomes shorter than the excursion time, $\tau < a^2/mr_C v_{Te}$, the "plateau" regime is realized, in which the thermal diffusivity becomes independent of the collision frequency, $\chi \sim (\varepsilon r_C v_{Te}/m)\left(a\delta B/r_C B_0\right)^2$.]

The presence of a radial electric field complicates the situation. Depending on its sign, the EXB drift is directed oppositely to the gradB drift for at least one of the particle species. As the EXB drift does not depend on particle energy, there would always exist a group of particles in the velocity space for which these two drifts cancel each other (we mean here toroidal drift averaged over a period of the poloidal motion). This leads to an increase of the neoclassical transport compared to the estimate (19).

One more factor has to be taken into account for the case where perturbations are created by a non-steady-state convection. If the characteristic correlation time is shorter than the time within which an electron makes a full radial excursion, the fluctuating nature of perturbations becomes important. This factor may both decrease the transport and increase it (the latter would occur for the electron whose drift velocity resonates with phase velocity of perturbations).

To mitigate effects of enhanced radial transport, one can add a weak regular toroidal magnetic field to the system. We denote this field by B_{t0}. It does not depend on the toroidal angle φ but may, generally speaking, depend on Φ and l. In order for this field to have a significant effect, it must satisfy the condition

$$B_{t0} >> \delta B, (r_C/a)B_0 \tag{20}$$

(the second of these two inequalities means that the toroidal velocity associated with the particle motion along the field line would exceed the toroidal drift velocity). At the same time, we do not want to make the system to become a more "traditional" confinement system of the type of a spheromak or RFP where the toroidal and poloidal fields are of the same order of magnitude. So, we assume that

$$B_{t0} << B_0. \tag{21}$$

The latter inequality shows that, to the first order, one can still use Eq. (4) for tracing the field lines, with the only difference being that now one has to put B_{t0} instead of δB_t in the denominator. This yields:

$$\frac{d\Phi}{d\varphi}=\frac{2\pi\oint R\delta B_n dl}{\oint \frac{B_{t0}dl}{RB_0}} \tag{22}$$

In this case the radial wandering of the field line becomes smaller, of the order of $a\delta B/B_{to}$.

In summary: In the closed-field-lines confinement systems where only poloidal magnetic field is present, the magnetic field structure is very sensitive to external magnetic perturbations, unless condition $\Delta\Phi$=0 (see Eq. (2)) holds identically, as it does, e.g., in the case of ideal MHD perturbations. In a more general case, even small perturbations may cause large radial excursions of magnetic field lines. Particle drifts and particle collisions cause neoclassical diffusion. This channel of radial transport can, however, be reduced by reducing the amplitude of large-scale field errors or by introducing relatively small regular toroidal magnetic field.

Work performed for U.S. DoE by UC LLNL under Contract W-7405-ENG-48 under contract W-7405-ENG-48 and by MIT and CU under contract DE-FG02-91ER54109.

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Figure captions

Fig. 1 a) Field lines of an axisymmetric current ring. Shaded are cross-sections of the conductor; R is the distance from the geometrical axis to a certain point on the unperturbed flux surface. For a finite plasma beta, the unperturbed field lines may differ substantially from those of a vacuum magnetic field. The coordinates l, Φ, φ are determined by the total unperturbed magnetic field (the one determined by the combination of external currents and plasma currents). The dash-dotted vertical line is the axis z. b) Puncture plot in the equatorial plane for a global (h~a) perturbation with toroidal mode number m=3.

Fig.2 a) Schematic of the magnetic configuration of the racetrack-type FRC. The field lines are straight everywhere except the end sections (shown in dashes). The outermost line represents the separatrix, whose distance from the axis is denoted as a. The dotted line represents a line where field reversal occurs. The distance of this line from the axis is $a/2^{1/2}$. The parameters r_e (r_i) represent the distance of the outer (inner, w.r.t. the field reversal surface) part of some field line from the axis. b) A sketch of puncture plots in the equatorial plane of FRC for the case where perturbation is a uniform magnetic field directed along the axis x. By using Eq. (9), one can project this puncture plot into the area of the reversed flux, $r<a/2^{1/2}$. The cross-section is enlarged by a factor 2 compared to the panel a.

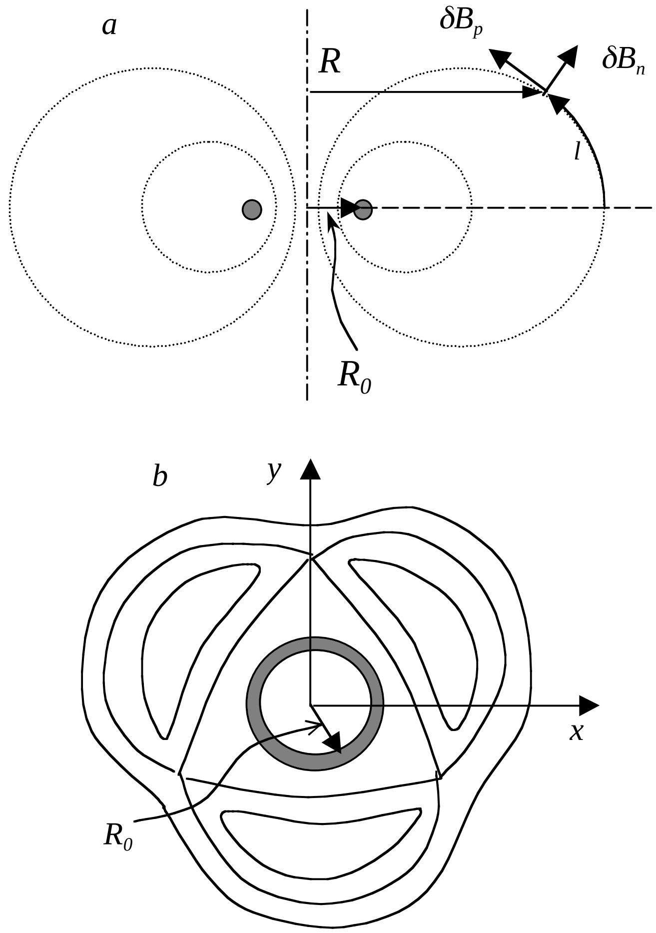

Fig.1

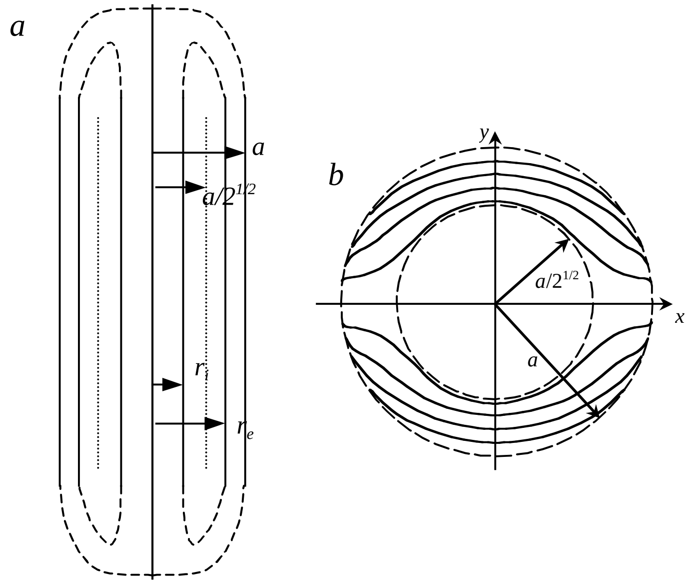

Fig. 2